Class: XII

SESSION : 2022-2023

SUBJECT: Mathematics

SAMPLE QUESTION PAPER - 2

with SOLUTION

Time Allowed: 3 Hours | **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. $\int_0^{\pi/2} \sin^2 x dx = ?$ **[1]**

a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$

c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$

2. Let P be a point on $\vec{r} = 5\hat{i} + 7\hat{j} - 2\hat{k} + s(3\hat{i} - \hat{j} + \hat{k})$ and Q be a point on the line $\vec{r} = -3\hat{i} + 3\hat{j} + 6\hat{k} + t(-3\hat{i} + 2\hat{j} + 4\hat{k})$. If $\overrightarrow{PQ}$ is parallel to the vector $2\hat{\mathrm{i}} + 7\hat{\mathrm{j}} - 5\hat{\mathrm{k}}$, then $|\overrightarrow{PQ}|$ equals: **[1]**

a) $2\sqrt{78}$ b) $\sqrt{78}$

c) $4\sqrt{78}$ d) $3\sqrt{78}$

3. The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ **[1]**

a) $\frac{128}{3}$ b) $\frac{64}{3}$

c) $\frac{32}{3}$ d) $\frac{256}{3}$

4. The area bounded by the curves $y = |x - 1|$ and $y = 1$ is given by **[1]**

a) 1 b) $\frac{1}{2}$

c) 2 d) none of these

5. If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors then $(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = ?$ **[1]**

a) 6 b) 12

c) 5 d) 3

6. Let A and B be two events such that P(A) = 0.6, P(B) = 0.2, and P(A | B) = 0.5. Then P (A′|B′) equals **[1]**

a) $\frac{3}{10}$

b) $\frac{6}{7}$

c) $\frac{3}{8}$

d) $\frac{1}{10}$

7. An unbiased die is tossed twice. What is the probability of getting a 4, 5 or 6 on the first toss and a 1, 2, 3 or 4 on the second toss? **[1]**

a) $\frac{2}{3}$

b) $\frac{3}{4}$

c) $\frac{1}{3}$

d) $\frac{5}{6}$

8. For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to **[1]**

a) $\overrightarrow{3a^2}$

b) $\overrightarrow{4a^2}$

c) $\overrightarrow{a^2}$

d) $\overrightarrow{2a^2}$

9. Maximise the function Z = 11x + 7y, subject to the constraints: x ≤ 3, y ≤ 2,x ≥ 0, y ≥ 0. **[1]**

a) 50

b) 48

c) 49

d) 47

10. General solution of $\frac{dy}{dx}$ + y tan x = sec x is: **[1]**

a) y tan x = sec x + c

b) y sec x = tan x + c

c) tan x = y tan x + c

d) x sec x = tan y + c

11. What is the solution of the differential equation $\frac{ydx-xdy}{y^2}=0$? **[1]**

a) x + y = C

b) x - y = C

c) xy = C

d) y = Cx

12. $\int\frac{dx}{\sqrt{2-4x+x^2}}$ =? **[1]**

a) $\log\left|(x-2)+\sqrt{x^2-4x+2}\right|+C$

b) $\log\left|(x+3)+\sqrt{x^2+6x+5}\right|+C$

c) $\log\left|x+\sqrt{x^2-4x+2}\right|+C$

d) None of these

13. The solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is **[1]**

a) $\phi\left(\frac{y}{x}\right)=ky$

b) $x\phi\left(\frac{y}{x}\right)=k$

c) $y\phi\left(\frac{y}{x}\right)=k$

d) $\phi\left(\frac{y}{x}\right)=kx$

14. If $A=\frac{1}{3}\begin{bmatrix}1 & 1 & 2\\ 2 & 1 & -2\\ x & 2 & y\end{bmatrix}$ satisfies $A^TA = I$, then x + y = **[1]**

a) -3

b) none of these

c) 0

d) 3

15. $\underset{x\to 0}{Lt}\ \frac{x(e^{\sin x}-1)}{1-\cos x}$ is equal to **[1]**

a) 2

b) 1

c) $\frac{1}{2}$

d) 0

16. The solution of $x\frac{dy}{dx}$ + y = e^x is: **[1]**

a) $x=\frac{e^y}{y}+\frac{k}{y}$

b) $y=\frac{e^x}{x}+\frac{k}{x}$

c) $y = xe^x + k$

d) $y = xe^x + cx$

17. If a line makes angles α, β and γ with the axes respectively, then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma\ =$ **[1]**

a) -2

b) 2

c) -1

d) 1

18. The value of sin (2 $\tan^{-1}$ (0.75)) is equal to **[1]**

a) 1.5

b) sin (1.5)

c) 0.75

d) 0.96

19. **Assertion (A):** Maximum value of Z = 3x + 2y, subject to the constraints x + 2y ≤ 2; x ≥ 0; y ≥ 0 will be obtained at point (2, 0). **[1]**
Reason (R): In a bounded feasible region, it always exist a maximum and minimum value.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** Every differentiable function is continuous but converse is not true. **[1]**
Reason (R): Function f(x) = |x| is continuous.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

Section B

21. Differentiate the function $\cos x \cdot \cos 2x \cdot \cos 3x$ w.r.t. x. **[2]**

22. Find the angle between the pairs of lines: **[2]**
$\frac{x-5}{1} = \frac{2y+6}{-2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$

OR

Find the cartesian and vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

23. A fair die is rolled. Consider events E = {1,3,5}, F = {2,3} and G = {2,3,4,5}. **[2]**
Find P(E|G) and P(G|E)

24. Solve: y dx - (x + 2y^2) dy = 0 **[2]**

25. For the principal values, evaluate $\sin^{-1}\left[\cos\left\{2cosec^{-1}(-2)\right\}\right]$ **[2]**

Section C

26. Solve the Linear Programming Problem graphically: **[3]**
Minimize Z = 2x + 4y Subject to
$x + y \geq 8$
$x + 4y \geq 12$
$x > 3, y \geq 2$

27. Evaluate: $\int \frac{1}{x(x^4-1)} dx$ **[3]**

OR

Evaluate: $\int \frac{2x+1}{\sqrt{x^2+2x-1}} dx$

28. Find the area of the region between the parabola x - y^2 - 6 y and the line x = -y. **[3]**

OR

Find the area bounded by the curve y^2 = 4a^2 (x - 1) and the lines x = 1 and y = 4a.

29. Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = - 1 and x = 1 **[3]**

30. Differentiate the function with respect to x: $\tan^{-1}\left(\frac{5x}{1-6x^2}\right), -\frac{1}{\sqrt{6}} < x < \frac{1}{\sqrt{6}}$. **[3]**

31. Find the angle between the lines whose direction cosines are given by the equations **[3]**
l + 2m + 3n = 0 and 3 Im - 4 In + mn = 0

OR

Find the angle between the lines whose direction ratios are 3, 2, -6 and 1, 2, 2.

Section D

32. Find the matrix A satisfying the matrix equation **[5]**

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix} A \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}.$$

OR

If A $= \begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$, find adj A and verify that A (adj A) = (adj A) A= |A| I_3.

33. If with reference to the right handed system of mutually $\perp$ unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ and $\vec{\alpha} = 3\hat{i} - \hat{j}, \vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}$ is || to $\vec{\alpha}$ and $\vec{\beta}_2$ is $\perp$ to $\vec{\alpha}$ **[5]**

34. Integrate the function: $\frac{1}{1-\tan x}$ **[5]**

35. Let A = R – {3} and B = R – {1}. Consider the function f: A $\Rightarrow$B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer. **[5]**

OR

Show that the function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, x $\in$ R is one-one and onto function.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) If unit sales prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively, then find the total revenue collected from Market-I &II.

(ii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find the cost price in Market I and Market II.

(iii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find gross profit from both the markets.

OR

If matrix A = $[a_{ij}]_{2\times 2}$ where $a_{ij} = 1$, if $i \neq j$ and $a_{ij} = 0$, if $i = j$ then find A^2.

37. **Read the text carefully and answer the questions:** **[4]**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is ₹100/m^2.

(i) If r cm be the radius and h cm be the height of the cylindrical tin can, then express the surface area as a function of radius (r)

(ii) Find the radius of the can that will minimize the cost of tin used for making can?

(iii) Find the height that will minimize the cost of tin used for making can ?

OR

Find the minimum cost of material used to manufacture the tin can.

38. **Read the text carefully and answer the questions:** **[4]**

Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(i) Find the probability that daughter is at one end, given that father and mother are in the middle.

(ii) Find the probability that mother is at right end, given that son and daughter are together.

SOLUTION

Section A

1. **(d)** $\frac{\pi}{4}$

Explanation: $I = \int_0^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} dx$

$= \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)_0^{\frac{\pi}{2}}$

$= \frac{\pi}{4} - \frac{\sin \pi}{4}$

$= \frac{\pi}{4}$

2. **(b)** $\sqrt{78}$

Explanation: $\sqrt{78}$

3. **(d)** $\frac{256}{3}$

Explanation: Since area $= 2\int_0^{16} \sqrt{y}dy$, solve the integral to compute the value.

4. **(a)** 1

Explanation: The given curves are : (i) $y = x - 1$, $x > 1$. (ii) $y = -(x - 1)$, $x < 1$. (iii) $y = 1$ these three lines enclose a triangle whose area is : $\frac{1}{2}$.base.height $= \frac{1}{2}.2.1 = 1$ sq. unit.

5. **(d)** 3

Explanation: 3

Given $|\vec{a}|^2 = |\vec{b}|^2 = 1$ and $\vec{a} \cdot \vec{b} = 0$

$(3\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 6\vec{b}) = 15|\vec{a}|^2 - 12|\vec{b}|^2 - 8\vec{a}.\vec{b}$

$= (15 \times 1) - (12 \times 1) - (8 \times 0)$

$= (15 - 12 - 0) = 3$

6. **(c)** $\frac{3}{8}$

Explanation: $P(A \cap B) = P(A | B) P(B)$

$= 0.5 \times 0.2 = 0.1$

$$P\left(A' | B'\right) = \frac{P\left(A' \cap B'\right)}{P\left(B'\right)} = \frac{P[(A \cup B)']}{1 - P(B)} = \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - 0.2} = \frac{3}{8}$$

7. **(c)** $\frac{1}{3}$

Explanation: Here S = {1, 2, 3, 4, 5, 6}. Let A = {4, 5, 6) and B = {1, 2, 3, 4}.

$$\therefore \quad P(A) = \frac{3}{6} = \frac{1}{2} \text{ and } P(B) = \frac{4}{6} = \frac{2}{3}$$

Clearly, A and B are independent events.

$$\therefore P(A \cap B) = P(A) \times P(B) = \left(\frac{1}{2} \times \frac{2}{3}\right)$$

$$= \frac{1}{3}.$$

8. **(d)** $2\vec{a}^2$

Explanation: Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then

$$\vec{a} \times \hat{i} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right) \times \hat{i} = a_1\hat{i} \times \hat{i} + a_2\hat{j} \times \hat{i} + a_3\hat{k} \times \hat{i}$$

$$\Rightarrow \vec{a} \times \hat{i} = 0 - a_2\hat{k} + a_3\hat{j}(\because \hat{i} \times \hat{i} = 0, \hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{i} = \hat{j})$$

$$\Rightarrow \vec{a} \times \hat{i} = -a_2\hat{k} + a_3\hat{j}$$

$$\Rightarrow |\vec{a} \times \hat{i}|^2 = a_2^2 + a_3^2$$

Similarly, we get

$$\Rightarrow |\vec{a} \times \hat{j}|^2 = a_1^2 + a_3^2$$

$$\Rightarrow |\vec{a} \times \hat{k}|^2 = a_1^2 + a_2^2$$

$$\therefore |\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = a_2^2 + a_3^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$$

$$= 2\left(a_1^2 + a_2^2 + a_3^2\right) = 2|\vec{a}|^2(\because |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}).$$

9. **(d)** 47

Explanation: We have , Maximise the function Z = 11x + 7y, subject to the constraints: $x \le 3$, $y \le 2, x \ge 0$, $y \ge 0$.

Corner points	Z = 11x +7 y
C(0, 0)	0
B (3,0)	33
D(0,2)	14
A(3, 2)	47

Hence the function has maximum value of 47

10. **(b)** y sec x = tan x + c

Explanation: We have, $\frac{dy}{dx} + y \tan x = \sec x$

which is a linear differential equation

Here, P = tan x, Q = sec x,

$$\therefore \quad \text{I.F} = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$$

$\therefore$ The general solution is

$y \sec x = \int \sec x \cdot \sec x + C$

$\Rightarrow$ $y \sec x = \int \sec^2 x dx + C$

$\Rightarrow$ $y \sec x = \tan x + C$

11. **(d)** $y = Cx$

Explanation: Consider the given differential equation,

$$\frac{ydx - xdy}{y^2} = 0 \Rightarrow d\left(\frac{x}{y}\right) = 0 \left[\because d\left(\frac{u}{v}\right) = \frac{v \cdot du - u \cdot dv}{v^2}\right]$$

On integrating both side, we get

$$\Rightarrow \int d\left(\frac{x}{y}\right) = C_1 \Rightarrow \frac{x}{y} = C_1 \Rightarrow y = Cx, \text{ where } C = \frac{1}{C_1}$$

12. **(a)** $\log|(x-2) + \sqrt{x^2 - 4x + 2}| + C$

Explanation: The given integral is $\int \frac{dx}{\sqrt{2 - 4x + x^2}}$

$$I = \int \frac{dx}{\sqrt{\left(x^2 - 4x + 4\right) - 2}} = \int \frac{dx}{\sqrt{(x-2)^2 - (\sqrt{2})^2}}$$

$$= \log|(x-2) + \sqrt{x^2 - 4x + 2}| + C$$

13. **(d)** $\phi\left(\frac{y}{x}\right) = kx$

Explanation: We have,

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)} \quad \text{....(i)}$$

Put $v = \frac{y}{x}$

$$\Rightarrow x\frac{dv}{dx} + v = \frac{dy}{dx}$$

$$\Rightarrow x\frac{dv}{dx} + v = v + \frac{\phi(v)}{\phi'(v)} \quad \text{.. from (i)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\phi(v)}{\phi'(v)}$$

$$\Rightarrow \frac{\phi'(v)}{\phi(v)} dv = \frac{dx}{x}$$

$$\Rightarrow \int \frac{\phi'(v)}{\phi(v)} dv = \int \frac{dx}{x}$$

$\Rightarrow$ log ϕ(v) = log |x|+log k

$$\Rightarrow \log \phi\left(\frac{y}{x}\right) - \log |x| = \log k$$

$$\Rightarrow \log\left[\frac{\phi\left(\frac{y}{x}\right)}{x}\right] = \log k$$

$$\Rightarrow \phi\left(\frac{y}{x}\right) = kx$$

14. **(b)** none of these

Explanation: We have, $A = \frac{1}{3}\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$

$$\Rightarrow A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & x \\ 1 & 1 & 2 \\ 2 & -2 & y \end{bmatrix}$$

Now, $A^TA = I$

$$\Rightarrow \begin{bmatrix} x^2+5 & 2x+3 & xy-2 \\ 3+2x & 6 & 2y \\ xy-6 & 2y & y^2+8 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

The corresponding elements of two equal matrices are not equal.
Thus, the matrix A is not orthogonal.

15. **(a)** 2

Explanation: $\lim\limits_{x \to 0} \frac{x(e^{\sin x} - 1)}{1 - \cos x}$

$$= \lim_{x \to 0} \frac{\frac{(e^{\sin x} - 1)}{x}}{\frac{1 - \cos x}{x^2}} = \lim_{x \to 0} \frac{(e^{\sin x} - 1)}{\sin x} \cdot \frac{\sin x}{x} .2 = 2\left(\because \lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}\right)$$

16. **(b)** $y = \frac{e^x}{x} + \frac{k}{x}$

Explanation: We have, $x\frac{dy}{dx} + y = e^x$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$

This is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$P = \frac{1}{x}$ and $Q = \frac{e^x}{x}$

$\therefore$ IF $= e^{\int \frac{1}{x} dx} = e^{(\log x)} = x$

So, the general solution is:

$y \cdot x = \int \frac{e^x}{x} x dx$

$\Rightarrow$ $y \cdot x = \int e^x dx$

$\Rightarrow$ $y \cdot x = e^x + k$

$\Rightarrow y = \frac{e^x}{x} + \frac{k}{x}$

17. **(c)** -1

Explanation: We know that, $Cos^2\alpha + Cos^2\beta + Cos^2\gamma = 1$.........(1)

Now, $Cos2\alpha + Cos2\beta + Cos2\gamma$

$= (2Cos^2\alpha - 1) + (2Cos^2\beta - 1) + (2Cos^2\gamma - 1)$

$= 2(Cos^2\alpha + Cos^2\beta + Cos^2\gamma) - 3$

$= 2\times1 - 3$ [from equation (1)]

$= -1$.

18. **(d)** 0.96

Explanation: $\sin(2\tan^{-1}(0.75))$

Let, $\tan^{-1}(0.75) = \theta$

$\Rightarrow$ $\tan^{-1}\left(\frac{3}{4}\right) = \theta$

$\Rightarrow$ $\tan\theta = \frac{3}{4}$

As, $\tan\theta = \frac{3}{4}$, so

$\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ ....(1)

Now,

$\sin(2\tan^{-1}(0.75)) = \sin 2\theta$

$= 2 \sin \theta \cos \theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{25}$

So, $\sin(2\tan^{-1}(0.75)) = 0.96$

19. **(b)** Both A and R are true but R is not the correct explanation of A.

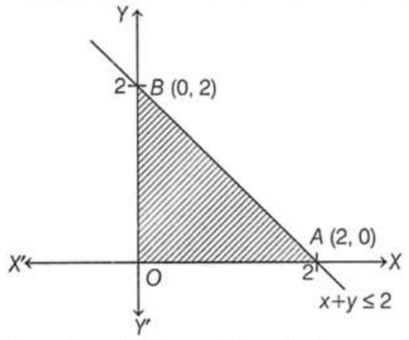
Explanation: Assertion: Given, $x + y \leq 2, x \geq 0$ and $y \geq 0$

Let $Z = 3x + 2y$

Now, table for $x + y = 2$

x	0	2	1
y	2	0	1

At (0, 0), $0 + 0 \leq 2 \Rightarrow 0 \leq 2$, which is true.



So, shaded portion is towards the origin.

$\therefore$ The corner points of shaded region are O(0, 0), A(2, 0) and B(0, 2)

At point O(0, 0), $Z = 3(0) + 2(0) = 0$

At point A(2, 0), $Z = 3(2) + 2(0) = 6$

At point B(0, 2), $Z = 3(0) + 2(2) = 4$

Hence, maximum value of Z is 6 at point (2, 0).

Hence both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

20. **(c)** A is true but R is false.

Explanation: Assertion: It is a true statement.

Reason: We have, $f(x) = |x|$

At $x = 0$,

$$\text{LHL} = \lim_{h \to 0^-} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0^-} \frac{|0-h| - 0}{-h}$$

$$= \lim_{h \to 0^-} \frac{h}{-h} = -1$$

$$\text{and RHL} = \lim_{h \to 0^+} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0^+} \frac{|0+h|-0}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$$

Here, LHD $\neq$ RHD, hence f(x) is not continuous at x = 0.

Section B

21. Given function is: $cosx.\ cos2.cos3x$

Let y = $cosx.\ cos2.cos3x$

Taking log on both sides, we get

log y = log(cos x. cos 2x. cos 3x)

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log y) = \frac{d}{dx}\log(\cos x) + \frac{d}{dx}\log(\cos 2x) + \frac{d}{dx}(\log\cos 3x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}\cdot\frac{d}{dx}(\cos x) + \frac{1}{\cos 2x}\cdot\frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x}\cdot\frac{d}{dx}(\cos 3x)$$

$$\Rightarrow \frac{dy}{dx} = y\left[-\frac{\sin x}{\cos x} - \frac{\sin 2x}{\cos 2x}\cdot\frac{d}{dx}(2x) - \frac{\sin 3x}{\cos 3x}\cdot\frac{d}{dx}(3x)\right]$$

$$\Rightarrow \frac{dy}{dx} = -\cos x\cdot\cos 2x\cdot\cos 3x[\tan x + \tan 2x(2) + \tan 3x(3)]$$

$$\Rightarrow \frac{dy}{dx} = -\cos x\cdot\cos 2x\cdot\cos 3x[\tan x + 2\tan 2x + 3\tan 3x]$$

22. In the Cartesian or symmetrical form of equation the angle between two lines can be found by dot product equation and in this equation we will use the direction ratios which are in the denominator of the equation.

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

In this equation a,b,c are the direction ratios of this equation.

Equations of the given lines are,

$$\frac{x-5}{1} = \frac{2y+6}{-2} = \frac{z-3}{1} \text{ and } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$$

These are not in the standard form but after converting them, we get,

$$\frac{x-5}{1} = \frac{y+3}{-1} = \frac{z-3}{1} \text{ and } \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-6}{5}$$

Given there are two vectors which are parallel to these lines,

$\vec{a} = 1\hat{i} - 1\hat{j} + 1\hat{k}, \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

So angle between the lines is the angle between these vectors which are parallel to the lines.

By using dot product equation,

$$\cos\theta = \frac{\vec{a}\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

$$\cos\theta = \frac{(1\hat{i}-1\hat{j}+1\hat{k})\,(3\hat{i}+4\hat{j}+5\hat{k})}{|1\hat{i}-1\hat{j}+1\hat{k}|\,|3\hat{i}+4\hat{j}+5\hat{k}|}$$

$$\cos\theta = \frac{1\times(3)+(-1)\times 4+1\times 5}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+5^2}}$$

$$\cos\theta = \frac{3-4+5}{\sqrt{3}5\sqrt{2}}$$

$$\cos\theta = \frac{4}{5\sqrt{6}}$$

$$\theta = \cos^{-1}\left(\frac{4}{5\sqrt{6}}\right)$$

Which is the required angle between the lines

OR

The given line is

$$\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$$

or $\frac{x+2}{-1} = \frac{y+3}{7} = \frac{z-3}{\frac{3}{2}}$ or $\frac{x+2}{-2} = \frac{y+3}{14} = \frac{z-3}{3}$

Therefore direction ratios are -2, 14, 2

$\therefore$ Cartesian equation of line through (1, 2, 3) and having direction ratios -2, 14, 3 are

$$\frac{x-1}{-2} = \frac{y-2}{14} = \frac{z-3}{3}$$

Now, $\vec{a} = \hat{i} + 2\hat{j} + 3\vec{k}, \vec{m} = -2\hat{i} + 14\hat{j} + 3\hat{k}$

$\therefore$ Hence the required Vector equation of line is

$\vec{r} = \vec{a} + \lambda\vec{m}$

or $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-2\hat{i} + 14\hat{j} + 3\hat{k})$

23. Sample space for the given experiment, 'S' = {1, 2, 3, 4, 5, 6}

Here, E = {1, 3, 5}, F = {2, 3} and G = {2, 3, 4, 5} .....(i)

$\Rightarrow P(E) = \frac{3}{6} = \frac{1}{2}, P(F) = \frac{2}{6} = \frac{1}{3}, P(G) = \frac{4}{6} = \frac{2}{3}$ ......(ii)

Now, $E \cap F = \{3\}, F \cap G = \{2, 3\}, E \cap G = \{3, 5\}$ .....(iii)

$\Rightarrow P(E \cap F) = \frac{1}{6}, P(F \cap G) = \frac{2}{6} = \frac{1}{3}, P(E \cap G) = \frac{2}{6} = \frac{1}{3}$ ......(iv)

By definition of conditional probability, $P(E|F) = \frac{P(E\cap F)}{P(F)}$

$$\Rightarrow P(E|G) = \frac{P(E\cap G)}{P(G)} = \frac{1/3}{2/3} = \frac{1}{2}$$

$$\Rightarrow P(E|G) = \frac{1}{2}$$

Similarly, we have

$$P(G|E) = \frac{P(G \cap E)}{P(E)} = \frac{1/3}{1/2} = \frac{2}{3}$$

$$\Rightarrow P(G|E) = \frac{2}{3}$$

24. The given differential equation is y dx - (x + 2y^2) dy = 0

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x+2y^2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{x+2y^2}{y}$$

$$\Rightarrow \frac{dx}{dy} + \left(-\frac{1}{y}\right)x = 2y \text{ ...(i)}$$

This is a linear differential equation of the form

$\frac{dx}{dy}$ + Px = Q, where P = $-\frac{1}{y}$ and Q = 2y

$$\therefore \text{ I.F. } = e^{\int \text{Rdy}} = e^{\int \frac{-1}{y} dy} = e^{-\log y} = e^{\log y^{-1}} = y^{-1}$$

Multiplying both sides of (i) by I.F. = y^{-1}, we obtain

$$\frac{1}{y}\frac{dx}{dy} - \frac{1}{y^2}x = 2$$

Integrating both sides with respect to y, we get

$x \times \frac{1}{y}$ = ∫ 2 dy + C [Using: x (I.F.) = ∫ Q (I.F.) dy + C]

$\Rightarrow \frac{x}{y}$ = 2y + C, which is the required solution to given differential equation.

25. First of all we need to find the principal value for cosec^{-1}(–2)

Let,

cosec^{-1}(-2)=Y

$\Rightarrow$ cosec y = –2

$\Rightarrow$ –cosec y = 2

$\Rightarrow$ –cosec $\frac{\pi}{6}$ = 2

As we know that cosec($-\theta$) = –cosecθ

$\therefore$ –cosec $\frac{\pi}{6}$ = cosec $\left(\frac{-\pi}{6}\right)$

The range of principal value of cosec^{-1} (-2) is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$–{0} and

$\text{cosec}\left(\frac{-\pi}{6}\right) = -2$

Thus,the princi value of $\text{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

$\therefore$ Now, the question changes to

$\text{Sin}^{-1}[\cos\frac{-\pi}{6}]$

$\text{Cos}(-\theta) = \cos(\theta)$

$\therefore$ we can write the above expression as

$\text{Sin}^{-1}[\cos\frac{\pi}{6}]$

Let,

$\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = Y$

$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$

$\Rightarrow Y = \frac{\pi}{3}$

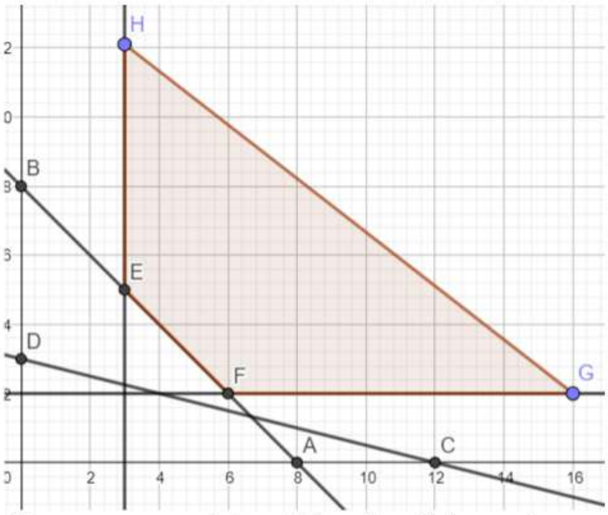
The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

Hence, the principal value of the given equation is $\frac{\pi}{3}$

Section C

26. First, we will convert the given inequations into equations, we obtain the following equations:
$x + y = 8$, $x + 4y = 12$, $x = 3$, $y = 2$ and solving we get values are as follows:
The region represented by $x + y \geq 8$: The line $x + y = 8$ meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line $x + y = 8$ Clearly (0,0) does not satisfies the inequation $x + y \geq 8$. So, the region in x y plane which does not contain the origin represents the solution set of the inequation $x + y \geq 8$
The region represented by $x + 4y \geq 12$:
The line $x + 4y = 12$ meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line $x + 4y = 12$.
Clearly (0,0) satisfies the inequation $x + 4y \geq 12$. So, the region in x y plane which contains the origin represents the solution set of the inequation $x + 4y \geq 12$
The line $x = 3$ is the line that passes through the point (3,0) and is parallel to Y axis $x \geq 3$ is the region to the right of the line $x = 3$
The line $y = 2$ is the line that passes through the point (0,12) and is parallel to X axis.$y \geq 2$ is the region above the line $y = 2$



The corner points of the feasible region are E(3,5) and F(6,2)
The values of Z at these corner points are as follows.

Corner point	Z = 2x + 4y
E(3, 5)	$2 \times 3 + 4 \times 5 = 26$
F(6, 2)	$2 \times 6 + 4 \times 2 = 20$

Therefore, the minimum value of objective function Z is 20 at point F(6,2). Hence, x = 6 and y = 2 is the optimal solution of the given LPP. Thus, the optimal value of objective function z is 20.

27. Let, $I = \int \frac{1}{x\left(x^4 - 1\right)} dx$

Using partial fraction,

$$\frac{1}{x\left(x^4-1\right)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{x^2+1}$$

$1 = A(x + 1)(x - 1)(x^2 + 1) + Bx(x - 1)(x^2 + 1) + Cx(x + 1)(x^2 + 1) + Dx(x + 1)(x - 1)$

For x = 0, A = -1,

For x = 1, $C = \frac{1}{4}$

For x = -1, $B = \frac{1}{4}$

For x = 2, $D = \frac{1}{4}$

Therefore

$$\int \frac{1}{x\left(x^4-1\right)} dx = -\int \frac{1}{x} dx + \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int \frac{dx}{x^2+1}$$

$$= -\ln|x| + \frac{1}{4}\ln|(x+1)| + \frac{1}{4}\ln(x-1)\Big| + \frac{1}{4}\ln\Big|\left(x^2+1\right)| + c$$

$$= \frac{1}{4}\ln\left|\frac{x^4-1}{x^4}\right| + c$$

OR

Let the given integral be,

$$I = \int \frac{2x+1}{\sqrt{x^2+2x-1}}dx$$

Let $2x + 1 = \lambda\frac{d}{dx}(x^2 + 2x - 1) + \mu$

$= \lambda(2x + 2) + \mu$

$2x + 1 = (2\lambda)x + 2\lambda + \mu$

Comparing the coefficients of like powers of x,

$2\lambda - 1 \Rightarrow \lambda = 1$

$2\lambda + \mu = 1 \Rightarrow 2(1) + \mu = 1$

$\mu = -1$

So, $I = \int \frac{(2x+2)-1}{\sqrt{x^2+2x-1}}dx$

$$= \int \frac{(2x+2)}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{x^2+2x+(1)^2-(1)^2-1}}dx$$

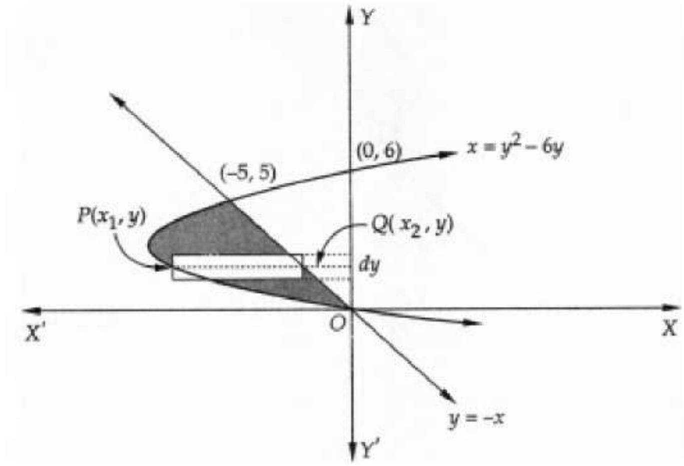
$$I = \int \frac{2x+2}{\sqrt{x^2+2x-1}}dx - \int \frac{1}{\sqrt{(x+1)^2-(\sqrt{2})^2}}dx$$

$I = (2\sqrt{x^2+2x-1}) - \log|(x+1) + \sqrt{(x+1)^2-(\sqrt{2})^2}| + c$ [since, $\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x} + c$,

$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log|x + \sqrt{x^2-a^2}| + c$]

$I = 2\sqrt{x^2+2x-1} - \log|x + 1 + \sqrt{x^2+2x-1}| + c$

28. he equation $x = y^2 - 6y$ can be written as $(y - 3)^2 = x + 9$. Clearly, it represents the parabola having vertex at (-9, 3) and opens rightward. The sketch of the parabola is shown in Fig. The equation $y = -x$ represents a line passing through the origin making 135° angle with x-axis. To find the points of intersection of these two curves, we solve the equations $x = y^2 - 6y$ and $x = -y$ simultaneously.



Putting $x = -y$ in $x = y^2 - 6y$, we get

$y^2 - 6y = -y \Rightarrow y(y - 5) = 0 \Rightarrow y = 0$ and $y = 5$

Putting $y = 0$ and $y = 5$ in $y = -x$ respectively, we obtain $x = 0$ and $x = -5$ respectively. Thus, the parabola and the line intersect at O (0, 0) and (-5, 5). The region enclosed by the two curves is shaded in the above Figure .Let us slice this region into horizontal strips. The approximating rectangle shown in Figure. has length $= \left|x_2 - x_1\right|$ width = dy and, area $= \left|x_2 - x_1\right|$ dy. Clearly, it can move vertically between $y = 0$ and $y = 5$. So, the required area of the shsded region denoted by A ,is given by

$$A = \int_0^5 \left|x_2 - x_1\right| dy$$

$$\Rightarrow A = \int_0^5 \left(x_2 - x_1\right) dy \left[\because x_2 > x_1 \therefore \left|x_2 - x_1\right| = x_2 - x_1\right]$$

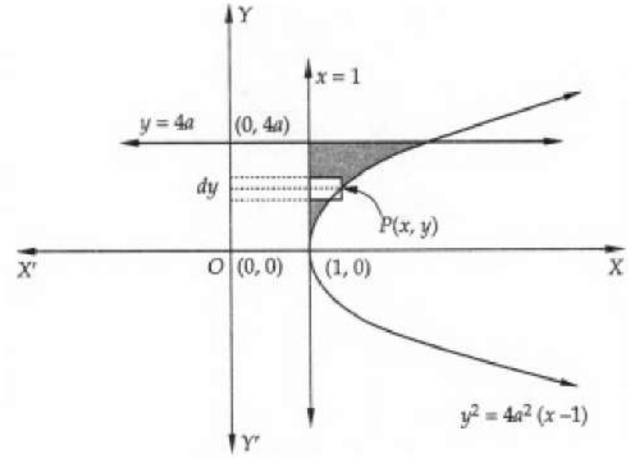
$\Rightarrow A = \int_0^5 \left\{-y - \left(y^2 - 6y\right)\right\} dy$ [Because $Q(x_2, y)$ and $P(x, y)$ lie on $y = -x$ & $x = y^2 - 6y$ respectly . Also $x_2 = -y$ and $x_1 = y^2 - 6y$]

$$\Rightarrow A = \int_0^5 \left(5y - y^2\right) dy = \left[\frac{5}{2}y^2 - \frac{y^3}{3}\right]_0^{-5} = \frac{125}{6} \text{ sq. units}$$

OR

The equation of the given curve is $y^2 = 4a^2 (x - 1)$ or, $(y - 0)^2 = 4a^2 (x - 1)$.

Clearly, this equation represents a parabola with vertex at (1, 0) as shown in Fig. The region enclosed by $y^2 = 4a^2 (x - 1)$, $x = 1$ and $y = 4a$ is the area of shaded portion in Fig.



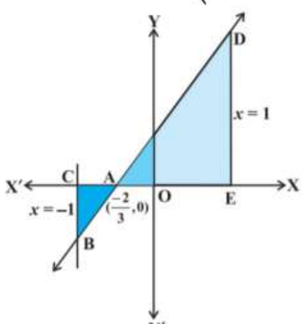
When we slice the area of the shaded portion in horizontal strips, we observe that each strip has left end on the line x = 1 and the right end on the parabola $y^2 = 4a^2 (x - 1)$. So, the approximating rectangle shown in Fig. has, length = x - 1, width = dy and area =(x -1) dy. Since, the approximating rectangle can move from y = 0 to y = 4a. So, required area denoted by A, of the shaded Region is given by

$$A = \int_0^{4a} (x-1)dy$$

$$\Rightarrow A = \int_0^{4a} \frac{y^2}{4a^2} dy \text{ [Because P(x, y) lies on } y^2 = 4a^2 (x - 1) \text{, since , } x - 1 = \frac{y^2}{4a^2}$$

$$\Rightarrow A = \frac{1}{4a^2}\left[\frac{y^3}{3}\right]_0^{4a} = \frac{1}{4a^2}\left(\frac{64a^3}{3}\right) = \frac{16a}{3} \text{ sq. units}$$

29. As shown in the fig., the line y = 3x + 2 meets x - axis A= $\frac{-2}{3}$ and its graph lies below x - axis for $x \in \left(-1, \frac{-2}{3}\right)$ and above x - axis for $x \in \left(\frac{-2}{3}, 1\right)$



The required area = Area of the region ACBA + Area of the region ADEA

$$= \left|\int_{-1}^{\frac{-2}{3}} (3x+2)dx\right| + \int_{\frac{-2}{3}}^{1} (3x+2)dx$$

$$= \left|\left[\frac{3x^2}{2} + 2x\right]_{-1}^{\frac{-2}{3}}\right| + \left[\frac{3x^2}{2} + 2x\right]_{\frac{-2}{3}}^{1} = \frac{1}{6} + \frac{25}{6} = \frac{13}{3}$$

30. Let, $y = \tan^{-1}\left(\frac{5x}{1-6x^2}\right)$

$\Rightarrow y = \tan^{-1}\left[\frac{3x+2x}{1-(3x)(2x)}\right]$

$\Rightarrow$ y = $\tan^{-1}(3x) + \tan^{-1}(2x)$ $\left[\text{Since}, \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

Differentiate it with respect to x using chain rule,

$\frac{dy}{dx} = \frac{1}{1+(3x)^2}\frac{d}{dx}(3x) + \frac{1}{1+(2x)^2}\frac{d}{dx}(2x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+9x^2} \times 3 + \frac{1}{1+4x^2} \times 2$

$\therefore \frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$

The differentiation of the given function y is as above.

31. According to question;

l + 2m + 3n = 0 ......(1)

3lm - 4 ln + mn = 0 ......(2)

From (1), we get

l = -2m - 3n

Substituting l = -2m -3n in (2), we get

3(-2m - 3n) m - 4 (-2m - 3n) n + mn = 0

$\Rightarrow -6m^2 - 9mn + 8mn + 12n^2 + mn = 0$

$\Rightarrow 12n^2 - 6m^2 = 0$

$\Rightarrow m^2 = 2n^2$

$\Rightarrow m = \sqrt{2}n, -\sqrt{2}n$

If m = $\sqrt{2}n$, then by substituting m = $\sqrt{2}n$ in (1) we get l = n (-2$\sqrt{2}$ - 3).

If m = -$\sqrt{2}n$, then by substituting m = -$\sqrt{2}n$ in (1) we get l = n (-2$\sqrt{2}$ - 3).

Thus, the direction ratios of the two lines are proportional to

n(-2$\sqrt{2}$ - 3), $\sqrt{2}n$, n and n (2$\sqrt{2}$ -3), $-\sqrt{2}n$, n

or $(-2\sqrt{2}-3)$, $\sqrt{2}$, 1 and $(-2\sqrt{2}-3)$ - $\sqrt{2}$.

Vectors parallel to these lines are

$\vec{a} = (-2\sqrt{2}-3)\hat{i} + \sqrt{2}\hat{j} + \hat{k}$

$\vec{b} = (2\sqrt{2}-3)\hat{i} - \sqrt{2}\hat{j} + \hat{k}$

If θ is the angle between the lines, then it is also the angle between $\vec{a}$ and $\vec{b}$.

Now,

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$

$$= \frac{[(-2\sqrt{2}-3)\hat{i}+\sqrt{2}\hat{j}+\hat{k}]\cdot[(2\sqrt{2}-3)\hat{i}-\sqrt{2}\hat{j}+\hat{k}]}{\sqrt{8+9+12\sqrt{2}+2+1}\sqrt{8+9-12\sqrt{2}+2+1}}$$

$$= \frac{-(8-9)-2+1}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$

$$= \frac{0}{\sqrt{20+12\sqrt{2}}\sqrt{20-12\sqrt{2}}}$$

$$= 0$$

$$\Rightarrow \theta = \frac{\pi}{2}$$

OR

Let the given lines be L_1 and L_2 respectively. Then,

D.r.'s of L_1 are 3, 2, -6.

And, $\sqrt{3^2+2^2+(-6)^2} = \sqrt{9+4+36} = \sqrt{49} = 7$

$\therefore$ d.c.'s of L_1 are $\frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$

D.r.'s of L_2 are 1, 2, 2

And $\sqrt{1^2+2^2+2^2} = \sqrt{9} = 3$

$\therefore$ d.r.'s of L_2 are $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

Suppose θ be the angle between L_1 and L_2 Then,we have

$\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$

$$= \left|\left(\frac{3}{7}\times\frac{1}{3}\right)+\left(\frac{2}{7}\times\frac{2}{3}\right)+\left(\frac{-6}{7}\times\frac{2}{3}\right)\right|$$

$$= \left|\frac{1}{7}+\frac{4}{21}-\frac{4}{7}\right| = \left|\frac{-5}{21}\right| = \frac{5}{21}$$

$$\theta = \cos^{-1}\left(\frac{5}{21}\right)$$

Section D

32. We have, $\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}_{2\times2} A \begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix}_{2\times2} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}_{2\times2}$

Let $A = \begin{bmatrix}a & b\\c & d\end{bmatrix}$

$$\therefore \begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}-3 & 2\\5 & -3\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a+6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ -6a - 3c + 10b + 5d = 1 ....(i)

$\Rightarrow$ 4a + 2c - 6b - 3d = 0 ....(ii)

$\Rightarrow$ -9a - 6c + 15b + 10d = 0 ....(iii)

$\Rightarrow$ 6a + 4c - 9b - 6d = 1 ....(iv)

On adding Eqs. (i) and (iv), we get

c + b - d = 2 $\Rightarrow$ d = c + b - 2 ....(v)

On adding Eqs. (ii) and (iii), we get

-5a - 4c + 9b + 7d = 0 ....(vi)

On adding Eqs. (vi) and (iv), we get

a + 0 + 0 + d = 1 $\Rightarrow$ d = 1 - a ....(vii)

From Eqs. (v) and (vii)

$\Rightarrow$ c + b - 2 = 1 - a $\Rightarrow$ a + b + c = 3 ....(viii)

$\Rightarrow$ a = 3 - b - c

Now, using the values of a and d in Eq. (iii) , we get

-9(3 - b - c) - 6c + 15b + 10(-2 + b + c) = 0

$\Rightarrow$ -27 + 9b + 9c - 6c + 15b - 20 + 10b + 10c = 0

$\Rightarrow$ 34b + 13c = 47

Now, using the values of a and d in Eq. (ii), we get

4(3 - b - c) + 2c - 6b - 3(b + c - 2) = 0

$\Rightarrow$ 12 - 4b - 4c + 2c - 6b - 3b - 3c + 6 = 0

$\Rightarrow$ -13b + 5c = 18 ....(x)

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{array}{c} -169b-65c=-234 \\ 170b+65c=235 \\ \hline b=1 \end{array}$$

$\Rightarrow$ -13 $\times$ 1 - 5c = -18 [from Eq. (x)]

$\Rightarrow$ -5c = -18 + 13 = -5 $\Rightarrow$ c = 1

a = 3 - 1 - 1 = 1 and d = 1 - 1 = 0

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

OR

Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly, the co-factors of elements of |A| are given by,

$A_{11} = \cos\alpha$; $A_{12} = -\sin\alpha$; $A_{13} = 0$;

$A_{21} = \sin\alpha$; $A_{22} = \cos\alpha$; $A_{23} = 0$

$A_{31} = 0$; $A_{32} = 0$ and $A_{33} = 1$

$$\therefore \text{ adj (A)} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, A (adj A)

$$= \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{...(i)}$$

$$(\text{ adj } A) \cdot (A) = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{...(ii)}$$

$$\text{and } |A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 1 \cdot (\cos^2\alpha + \sin^2\alpha) = 1$ [expanding along R_3]

$$\therefore \quad |A| I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From qs. (i), (ii) and (iii), we get,
A (adj A) = (adj A).A = |A| I_3

33. Let $\vec{\beta} = \lambda\vec{\alpha}$ $\left[\because \vec{\beta}_1 \parallel to\, \vec{\alpha}\right]$

$$\vec{\beta}_1 = \lambda(3\hat{i} - \hat{j})$$
$$= 3\lambda\hat{i} - \lambda\hat{j}$$
$$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$$
$$= (2\hat{i} + \hat{j} - 3\hat{k}) - (3\lambda\hat{i} - \lambda\hat{j})$$
$$= (2 - 3\lambda)i + (1 + \lambda)\hat{j} - 3\hat{k}$$
$$\vec{\alpha}.\vec{\beta}_2 = 0 \left[\because \vec{\beta}_2 \perp \vec{\alpha}\right]$$
$$3(2 - 3\lambda) - (1 + \lambda) = 0$$
$$\lambda = \frac{1}{2}$$
$$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$$
$$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$$

34. Let $I = \int \frac{1}{1 - \tan x} dx$

$$= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx$$
$$= \int \frac{1}{\left(\frac{\cos x - \sin x}{\cos x}\right)} dx$$
$$= \int \frac{\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{\cos x + \cos x}{\cos x - \sin x} dx$$

Adding and subtracting sin x in the numerator,

$$= \frac{1}{2}\int \frac{\cos x - \sin x + \sin x + \cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\sin x + \cos x)}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \frac{\cos x - \sin x}{\cos x - \sin x} + \frac{\sin x + \cos x}{\cos x - \sin x} dx$$
$$= \frac{1}{2}\int \left(1 + \frac{\sin x + \cos x}{\cos x - \sin x}\right) dx$$

$$= \frac{1}{2}\left[\int 1dx - \int \frac{-\sin x - \cos x}{\cos x - \sin x}\right]dx$$

$$= \frac{1}{2}[x - \log|\cos x - \sin x|] + c \ [\because \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c]$$

35. A = R – {3} and B = R – {1} and $f(x) = \left(\frac{x-2}{x-3}\right)$

Let $x_1, x_2 \in$ A, then $f(x_1) = \frac{x_1-2}{x_1-3}$ and $f(x_2) = \frac{x_2-2}{x_2-3}$

Now, for $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-3}{x_2-3}$$

$$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$$

$$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$$

$$\Rightarrow -3x_1 - 2x_2 = -2x_1 - 3x_2$$

$$= x_1 = x_2$$

$\therefore f$ is one-one function.

Now $y = \frac{x-2}{x-3}$

$$\Rightarrow y(x - 3) = x - 2$$

$$\Rightarrow xy - 3y = x - 2$$

$$\Rightarrow x(y - 1) = 3y - 2$$

$$\Rightarrow x = \frac{3y-2}{y-1}$$

$$\therefore f\left(\frac{3y-2}{y-1}\right) = \frac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3} = \frac{3y-2-2y+2}{2y-2-3y+3} = y$$

$$\Rightarrow f(x) = y$$

Therefore, f is an onto function.

OR

f is one-one: For any x, y $\in$ R - {+1}, we have $f(x) = f(y)$

$$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$$

$$\Rightarrow xy + x = xy + y$$

$$\Rightarrow x = y$$

Therefore, f is one-one function.

If f is one-one, let y = R - {1}, then $f(x) = y$

$$\Rightarrow \frac{x}{x+1} = y$$

$\Rightarrow x = \dfrac{y}{1-y}$

It is cleat that x $\in$ R for all y = R - {1}, also x = $\neq$ -1

Because x = -1

$\Rightarrow \dfrac{y}{1-y} = -1$

$\Rightarrow$ y = -1 + y

which is not possible.

Thus for each R - {1} there exists $x = \dfrac{y}{1-y} \in$ R - {1} such that

$$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$$

Therefore f is onto function.

Section E

36. **Read the text carefully and answer the questions:**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) Let A be the 2 $\times$ 3 matrix representing the annual sales of products in two markets.

$$A = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{array}{l} Market\ I \\ Market\ II \end{array} \end{array}$$

Now, revenue = sale price $\times$ number of items sold

$$AB = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 25000+3000+18000 \\ 15000+30000+8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

Therefore, the revenue collected from Market I = ₹46000 and the revenue collected from Market II = ₹53000.

(ii) Let C be the column matrix representing cost price of each unit of products x, y, z.

$$C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} 20000+2000+9000 \\ 12000+20000+4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Cost price in Market I is ₹31000 and in Market is ₹36000.

(iii)Now, Profit matrix = Revenue matrix - Cost matrix

$\Rightarrow$ AB - AC

$$\Rightarrow \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets = ₹15000 + ₹17000 = ₹32000

OR

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = I$$

37. **Read the text carefully and answer the questions:**

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectors. The tin can is made to hold 3 litres of sanitizer or disinfector. The cost of material used to manufacture the tin can is ₹100/m^2.

(i) Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume of cylinder = 3l = 3000 cm^3 ($\because$ 1l = 1000 cm^3)

$$\Rightarrow \pi r^2 h = 3000 \Rightarrow h = \frac{3000}{\pi r^2}$$

Now, the surface area, as a function of r is given by

$$S(r) = 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r\left(\frac{3000}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{6000}{r}$$

(ii) Now, $S(r) = 2\pi r^2 + \frac{6000}{r}$

$$\Rightarrow S'(r) = 4\pi r - \frac{6000}{r^2}$$

To find critical points, put S'(r) = 0

$$\Rightarrow \frac{4\pi r^3 - 6000}{r^2} = 0$$

$$\Rightarrow r^3 = \frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$$

Also, $S''(r)\Big|_{r = \sqrt[3]{\frac{1500}{\pi}}} = 4\pi + \frac{12000 \times \pi}{1500}$

$= 4\pi + 8\pi = 12\pi > 0$

Thus, the critical point is the point of minima.

(iii)

The cost of material for the tin can is minimized when r = $\sqrt[3]{\frac{1500}{\pi}}$ cm and the height is

$$\frac{3000}{\pi\left(\sqrt[3]{\frac{1500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{1500}{\pi}} \text{ cm.}$$

OR

We have, minimum surface area = $\frac{2\pi r^3 + 6000}{r}$

$$= \frac{2\pi \cdot \frac{1500}{\pi} + 6000}{\sqrt[3]{\frac{1500}{\pi}}} = \frac{9000}{7.8} = 1153.84 \text{ cm}^2$$

Cost of 1 m^2 material = ₹100

$\therefore$ Cost of 1 cm^2 material = ₹ $\frac{1}{100}$

$\therefore$ Minimum cost = ₹$\frac{1153.84}{100}$ = ₹11.538

38. **Read the text carefully and answer the questions:**
Family photography is all about capturing groups of people that have family ties. These range from the small group, such as parents and their children. New-born photography also falls under this umbrella. Mr Ramesh, His wife Mrs Saroj, their daughter Sonu and son Ashish line up at random for a family photograph, as shown in figure.

(i) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24
Let A denotes the event that daughter is at one end n(A) = 12 and B denotes the event that father, and mother are in the middle n(B) = 4
Also, n(A ∩ B) = 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{4}{24}} = 1$$

(ii) Sample space is given by {MFSD, MFDS, MSFD, MSDF, MDFS, MDSF, FMSD, FMDS, FSMD, FSDM, FDMS, FDSM, SFMD, SFDM, SMFD, SMDF, SDMF, SDFM DFMS, DFSM, DMSF, DMFS, DSMF, DSFM}, where F, M, D and S represent father, mother, daughter and son respectively. n(S) = 24
Let A denotes the event that mother is at right end. n(A) = 6 and B denotes the event that son and daughter are together. n(B) = 12
Also, n (A ∩ B) = 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{24}}{\frac{12}{24}} = \frac{1}{3}$$